

# Space-Time, General Covariance, Dirac-Bergmann Observables and Non-Inertial Frames.

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Both particle physics and all the approaches to gravity make use of variational principles employing singular Lagrangians [1]. As a consequence the Euler-Lagrange equations cannot be put in normal form, some of them may be non independent equations (due to the contracted Bianchi identities) and a subset of the original configuration variables are left completely or partially indeterminated. Moreover, even with reasonable boundary conditions at spatial infinity <sup>1</sup>, the fact that often Euler-Lagrange equations, like in the case of Einstein's equations, are not a hyperbolic system of partial differential equations implies a very difficult and nearly untractable initial value problem in configuration space.

This state of affairs is a source of an endless number of problems at the ontological level for philosophers of science. Instead in physics we accept the fact that the historical development of particle theory and of general relativity has selected certain variational principles and certain configuration variables as the most convenient to englobe useful properties like locality, manifest Lorentz covariance, minimal couplings (the gauge principle), general co-

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<sup>1</sup>We consider only non-compact spacetimes both in the special and general relativistic context, since a general relativistic description of particle physics must reduce to a field theory on Minkowski spacetime when the Newton constant  $G$  is switched off.

variance. However all this leads to the necessity of a division of the initial configuration variables of any theory in two groups:

- i) the *non determined gauge variables*;
- ii) the *gauge-invariant observables* with a deterministic evolution.

But this process is in conflict with locality, manifest Lorentz covariance, general covariance and, moreover, the configuration space manifestly covariant approach has no natural analytical tools to perform this separation. Even when this is possible, like in the radiation (or Coulomb) gauge for electromagnetism in Minkowski spacetime, we are not yet able to quantize, regularize and renormalize this formulation with only physical degrees of freedom since locality is lost <sup>2</sup>. Therefore particle physics in Minkowski spacetime relies on the strategy of quantizing all the configuration variables in an auxiliary unphysical Hilbert space  $\mathcal{H}_{unphys}$  and then in making a quantum reduction to a physical Hilbert space  $\mathcal{H}_{phys}$ , which, however, *in general is not a subspace* of  $\mathcal{H}_{unphys}$ . The winning strategy is the BRST approach <sup>3</sup> (with the BFM variant), but this is also the strategy of the group-theoretical, algebraic or geometric quantization approaches. Even if perturbative regularization and renormalization are well defined, BRST observables are well defined algebraic quantities and the phenomenological applications are successful, there is no clear hint of how to solve open problems like the determination of the *physical scalar product* of  $\mathcal{H}_{phys}$  and whether global obstructions like the Gribov ambiguity are either only mathematical obstructions, to be eliminated with a suitable choice of the function space for the fields, or carriers of physical information needed for instance to explain quark confinement. The alternative *first reduce, then quantize* is also very difficult to explore, because the classical reduced theory, when can be defined, lives usually in a topologically highly non-trivial configuration space. Incidentally, this is also the reason [3] why the path integral approach is not well defined

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<sup>2</sup>There is no no-go theorem, only the mathematical incapacity to treat the singularities [2].

<sup>3</sup>It puts control on the infinitesimal gauge transformations but not on the finite ones.

globally.

Another non trivial aspect of the need of the division between gauge variables and deterministic observables is the connection of the latter with measurable quantities. Since, at least at the classical level, the electromagnetic measurable quantities are the local electric and magnetic fields, we can extrapolate that the non-local radiation gauge observables, i.e. the transverse vector gauge potential and the transverse electric field, are also measurable. But in the case of the non-Abelian Yang-Mills gauge theories for the strong and weak interactions the connection between gauge invariant observables and measurable quantities is still poorly understood. With trivial principal bundles, in suitable weighted Sobolev spaces in which both the aspects of gauge symmetries and gauge copies of the Gribov ambiguity are absent, with all the fields tending suitably to zero in a direction-independent way so that the non-Abelian Yang-Mills charges are well defined, in a non-Abelian radiation gauge the non-local observables are again the transverse Yang-Mills vector gauge potentials and electric fields [4]. In more general cases it is unknown and we have only the algebraic BRST observables, which carry topological informations on the reduced configuration space but no global description of it, since perturbative methods cannot describe finite gauge transformations and the problems connected with the associated infrared divergencies [5].

When we come to general relativity in Einstein formulation these problems become both more complex and more basic. More complex because the Lie groups underlying the gauge groups of particle physics are replaced by diffeomorphism groups <sup>4</sup>, whose group manifold in large is poorly understood. More basic because now the action of the gauge group is not in an inner space of a field theory on a background spacetime, but is an extension to tensors over spacetime of the diffeomorphisms of the spacetime itself. This reflects itself in the much more

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<sup>4</sup>Reparametrization invariant theories in Minkowski spacetime for particles and strings and parametrized Minkowski theories for every isolated system [1,6] also have diffeomorphism groups as gauge groups.

singular nature of Einstein's equations <sup>5</sup> with respect to Yang-Mills equations. This fact has the dramatic consequence to *destroy any physical individuality of the points of spacetime* as evidenced by Einstein's *hole argument* [7] in the years (1913-16) of the genesis of the concept of general covariance. Only the idealization (*point-coincidence argument*) according to which all possible observations reduce to the intersections of the worldlines of observers, measuring instruments and measured physical objects, convinced Einstein to adopt general covariance and to abandon the physical objectivity of spacetime coordinates. This argument, after a long oblivion, was resurrected by Stachel [8] and then by Norton [9] and others as a basic problem [10] in our both ontological <sup>6</sup> and physical understanding of *spacetime* in general relativity.

General relativity is formulated starting from a mathematical pseudo-Riemannian 4-manifold  $M$  endowed with an atlas of coordinate charts. As a topological Hausdorff space, the points of  $M$  can be identified only by means of coordinates, i.e. quadruples of real numbers.

Einstein's equations are trivially form-invariant under *passive* diffeomorphisms, namely arbitrary changes of coordinates in  $M$ , being tensorial equations. Let us now consider *active* diffeomorphisms  $\mathcal{F} : M \mapsto M$ , under which a point  $P$  is sent into the point  $\mathcal{F}(P)$ . In a coordinate chart  $\mathcal{C}$ , if  $x^\mu(P)$  are the coordinates of  $P$ , then  $x^\mu(\mathcal{F}(P)) = f^\mu(x(P))$  are the coordinates of  $\mathcal{F}(P)$ . Given a 4-metric tensor  ${}^4g$  on  $M$  with components  ${}^4g_{\mu\nu}(x)$  in the chart  $\mathcal{C}$ , we can define a new metric tensor  ${}^4g'$  (the *drag-along* of  ${}^4g$ ) by asking that in

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<sup>5</sup>Four of them are not independent from the others due to the Bianchi identities, four are only restrictions on the initial data and only two combinations of Einstein's equations and their gradients depend on the *accelerations* (the second time derivatives of the metric tensor).

<sup>6</sup>See the modern debate [10] about the failure of *manifold substantivalism* to reconcile the need for determinism in classical physical laws and a *realistic* interpretation of the mathematical 4-manifold describing spacetime.

every coordinate chart containing  $P$  and  $\mathcal{F}(P)$  we have  ${}^4g'_{\mu\nu}(x(\mathcal{F}(P))) = {}^4g_{\mu\nu}(x(P))$ , so that  ${}^4g'_{\mu\nu}(x(P)) \neq {}^4g_{\mu\nu}(x(P))$ . If  ${}^4g$  is a solution of Einstein's equations, also  ${}^4g'$  is a solution, namely *active diffeomorphisms are dynamical symmetries* of Einstein's equations sending solutions into solutions.

Let us consider a *hole*  $\mathcal{A}$  in  $M$  not containing matter and an active diffeomorphism which is the identity outside  $\mathcal{A}$ , where all the matter resides and where suitable initial data and boundary conditions have been chosen. Then we have the situation that inside  $\mathcal{A}$  we have two different solutions  ${}^4g$  and  ${}^4g'$  which however coincide outside  $\mathcal{A}$ . Therefore there is no deterministic propagation of the solution from outside  $\mathcal{A}$  to the interior of  $\mathcal{A}$  and no way to attribute a physical individuality to the points of  $\mathcal{A}$  (which is the real gravitational field in a point?). The only way to avoid this lack of determinism is to say with Dirac that *only the equivalence class* of all the 4-metric tensors solution of Einstein's equations modulo active and passive diffeomorphisms is a *physical solution*. This property was named by Earman and Norton [11] *Leibniz equivalence* <sup>7</sup> and points toward the necessity of considering the 4-manifold  $M$  and the 4-metric tensor  ${}^4g$  <sup>8</sup> as an inseparable entity  $(M, {}^4g)$  in general relativity.

Let us remark that Bergmann and Komar [13] were able to give a passive re-interpretation

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<sup>7</sup>See Ref. [12] for the improper use of the name of Leibniz in a theory containing fields instead of only mechanical particles like in Newton-Leibniz times.

<sup>8</sup>It gives the *dynamical chronometrical structure* of  $M$ , differently from what happens in special relativity, where both Minkowski spacetime and the chronometrical structure (the clocks and the rods and therefore the whole theory of measurements) have a non-dynamical absolute existence. This also explains because till now we do not have a well defined theory of measurement in general relativity. As a consequence, there is a fundamental difference between generally covariant field theories like Einstein's general relativity and theories on a background like string theory. If  $\mathcal{M}$  theory exists and if its mathematical apparatus is not too exotic so that some form of general covariance can be defined, it too will have to face the *hole argument*.

of active diffeomorphisms, by showing that Einstein's equations are form-invariant not only under coordinate transformations  $x'^{\mu} = h^{\mu}(x)$ , but also under generalized transformations of the form  $x'^{\mu} = h^{\mu}(x, {}^4g(x))$ .

In physics the *hole argument* is considered an aspect of the fact that also Einstein's theory is interpreted as a gauge theory. The Leibnitz equivalence is nothing else than the selection of the gauge invariant observables of the theory. But now, differently from Yang-Mills theories, we have lost the physical interpretation of the underlying mathematical 4-manifold, and this suggests that a different interpretation of the gauge variables of generally covariant theories with respect to Yang-Mills theories is needed. Moreover, one would like that the observables in general relativity be *Bergmann observables* [14], namely (possibly local) quantities independent from the choice of coordinates. For instance the measurements of matter quantities in general relativity are idealized as performed by *test timelike observers* (either isolated or belonging to a congruence) *endowed with a tetrad* (the unit 4-velocity of the observer plus gyroscopes for the spatial triad): to avoid coordinate singularities and/or misunderstandings, the only acceptable measurable quantities are the *tetradic* components of matter tensors, which are Bergmann observables.

Let us remark that we do not accept the point of view <sup>9</sup> that matter is necessary for the physical individuation of spacetime points. The conceptual problem already exists in vacuum general relativity and has to be solved only by using the vacuum gravitational field, which has an ontological priority over all matter fields since it tells them how to move causally [16]. Test matter will only enter in the actual measurement process at an operational level.

As already said the manifestly covariant configuration space approach has no natural tool to make a clean separation between gauge variables and a basis of gauge invariant (hopefully measurable) observables. Instead, at least locally, the Hamiltonian formulation has natural tools for it, namely the Shanmugadhasan canonical transformations [17].

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<sup>9</sup>See the material reference fluids in Refs. [15].

The singular Lagrangians of particle physics and general relativity imply the use of Dirac-Bergmann theory [18,19,1] of Hamiltonian constraints<sup>10</sup> and only the constraint submanifold of phase space is relevant for physics. Let us consider a finite-dimensional system with configuration space  $Q$  with global coordinates  $q^i$ ,  $i = 1, \dots, N$  described by a singular Lagrangian  $L(q, \dot{q})$  [ $\dot{q}^i(\tau) = dq^i(\tau)/d\tau$ ]. Let the Dirac algorithm produce the following general pattern:

i)  $m < N$  first class constraints  $\phi_\alpha(q, p) \approx 0$ , of which the first  $m_1 \leq m$  are primary, with the property that the Poisson brackets of any two of them satisfies  $\{\phi_\alpha(q, p), \phi_\beta(q, p)\} = C_{\alpha\beta\gamma}(q, p) \phi_\gamma(q, p) \approx 0$  ;

ii)  $2n$  second class constraints, corresponding to pairs of canonical variables which can be eliminated by going to Dirac brackets;

iii) a Dirac Hamiltonian  $H_D = H_c + \sum_{\alpha=m_1+1}^m r_\alpha(q, p) \phi_\alpha(q, p) + \sum_{\alpha=1}^{m_1} \lambda_\alpha(\tau) \phi_\alpha(q, p)$ , where the  $\lambda_\alpha(\tau)$ 's are arbitrary functions of time, named *Dirac multipliers*, associated only with the *primary* first class constraints<sup>11</sup>. In phase space there will be as many arbitrary Hamiltonian gauge variables as first class constraints: they determine a coordinatization of the *gauge orbits* inside the constraint submanifold. The first class constraints are the generators of the Hamiltonian gauge transformations under which the theory is invariant and a gauge orbit is an equivalence class of all those configurations which are connected by

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<sup>10</sup>Namely of presymplectic geometry, the theory of a closed but degenerate two-form, in geometrical language.

<sup>11</sup>The use of the first half of Hamilton equations,  $\dot{q}^i = \{q^i, H_D\}$ , shows that the Dirac multipliers are those primary velocity functions ( $g_\alpha(q, \dot{q}) = \lambda_\alpha(\tau)$  on the solutions of Hamilton equations) not determined by the singular Euler-Lagrange equations. It can be shown that this arbitrariness implies that also the secondary velocity functions  $r_\alpha(q, p) = \tilde{r}_\alpha(q, \dot{q})$ ,  $\alpha = m_1 + 1, \dots, m$ , in front of the secondary (and higher) first class constraints in  $H_D$ , are not determined by the Euler-Lagrange equations. Therefore each first class constraint has either a configuration or a generalized velocity as an arbitrary partner.

gauge transformations (Leibnitz equivalence). The  $2(N - m - n)$ -dimensional reduced phase space is obtained by eliminating the second class constraints with Dirac brackets and by going to the quotient with respect to the gauge orbits, or equivalently by adding as many gauge fixing constraints as first class ones so to obtain  $2m$  second class constraints.

At least locally on the constraint submanifold the family of Shanmugadhasan canonical transformations  $q^i, p_i \mapsto Q^\alpha, P_\alpha \approx 0, \bar{Q}^\beta \approx 0, \bar{P}_\beta \approx 0, Q^A, P_A, \alpha = 1, \dots, m, \beta = 1, \dots, n$ , allows

i) to Abelianize the first class constraints, so that locally the constraint submanifold is identified by the vanishing of a subset of the new momenta  $P_\alpha \approx 0$ ;

ii) to identify the associated Abelianized gauge variables  $Q^\alpha$  as coordinates parametrizing the gauge orbits;

iii) to replace the second class constraints with pairs of canonical variables  $\bar{Q}^\beta \approx 0, \bar{P}_\beta \approx 0$ ;

iv) to identify a canonical basis of *gauge invariant Dirac observables* with a deterministic evolution determined only by the gauge invariant canonical part  $H_c$  of the Dirac Hamiltonian.

This is the tool of the Hamiltonian formalism, lacking in the configuration space approach, which allows to make the division between arbitrary gauge variables and deterministic gauge invariant observables.

Since the (in general non local) Dirac observables give a coordinatization of the classical reduced phase space, it will depend on its topological properties whether a given system with constraints admits a subfamily of Shanmugadhasan canonical transformations globally defined. When this happens the system admits *preferred* global separations between gauge and observable degrees of freedom. Therefore it is important to understand all the topological and/or singularity properties of the original configuration space, of the constraint submanifold and of the reduced phase space and to study how to solve the constraints, because otherwise there is no hope to get a global control on the system and one has only formal formulations of the dynamics.

In field theory the Shanmugadhasan canonical transformations are used in a heuristic way [1], because their existence has not yet been proved due to the fact that important



constraints like the Yang-Mills Gauss laws and the ADM supermomentum constraints are partial differential equations of elliptic type, which may admit zero modes according to the choice of the function space (see the Gribov ambiguity). In any case the identification of canonical bases of Dirac observables is so important for understanding the dynamics, that we can consider the assumption that certain Shanmugadhasan canonical transformations globally describe certain systems even in cases with non-trivial topology as a first approximation for extracting the main non-topological properties of a system [5].

In special relativity another source of complications originates from the fact that the constraint submanifold of an isolated system is the disjoint union of Poincare' strata [1,6]: all the configurations in each stratum have the conserved total 4-momentum belonging to a well defined type of Poincare' orbit. The main stratum contains all the timelike configurations with  $P^2 > 0$ . Therefore we have to define separate families of Shanmugadhasan canonical transformations for each stratum: they are not only adapted to the constraints but *also* to the little group of the Poincare' orbit. As a consequence, after a separate canonical reduction for each stratum, some of the Dirac observables will no more be manifestly Lorentz covariant, but they will only be covariant under the little group of the Poincare' orbit.

See Ref. [1] for a review of the special relativistic systems, including the  $SU(3) \times SU(2) \times U(1)$  particle standard model, to which this type of canonical reduction has been applied with the determination of the Dirac observables.

To get a universal control on this breaking of Lorentz covariance and to put all special relativistic isolated systems in a form oriented to the coupling to the gravitational field, we followed the indications of Dirac [18] to reformulate [1,6,20–23] all isolated systems on arbitrary (*simultaneity and Cauchy*) spacelike hypersurfaces, leaves of the foliation associated to a 3+1 splitting of Minkowski spacetime <sup>12</sup>. The embeddings  $z^\mu(\tau, \vec{\sigma})$  ( $\tau$  and  $\vec{\sigma}$  are

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<sup>12</sup>It is the classical basis of Tomonaga-Schwinger quantum field theory, in which the classical fields acquire the non-local information about the *equal-time* surfaces, information which is absent in the

coordinates adapted to the foliation with the scalar time parameter labeling the leaves) of these hypersurfaces  $\Sigma_\tau$  are *new configuration variables* and the induced metric on the hypersurfaces is a function of the embeddings. If we take the Lagrangian of the system coupled to an external gravitational field and we replace the 4-metric with the metric induced by the embedding, we get the singular Lagrangian for the embeddings plus the system (see Ref. [1,6] and Appendix A of Ref. [24]). These *parametrized Minkowski theories* have first class constraints (corresponding to the superhamiltonian and supermomentum ones of ADM canonical gravity and resulting from a deparametrization of general relativity), which imply that the description is *independent* from the choice of the 3+1 splitting. As a consequence, the embeddings  $z^\mu(\tau, \vec{\sigma})$  are the *gauge variables of this special relativistic type of general covariance*. Since, given the embeddings, the field of unit normals to the hypersurfaces and the evolution vector  $\partial_\tau z^\mu(\tau, \vec{\sigma})$  give rise to two (non-rotating the former, rotating the latter) congruences of timelike observers, parametrized Minkowski theories are also theories *describing timelike arbitrarily accelerated observers*, where to learn how to study special relativistic systems in *non-inertial* reference frames. Therefore, the Shanmugadhsan canonical transformations and the canonical reductions must be reformulated on arbitrary spacelike hypersurfaces. In special relativity the foliations with spacelike *hyperplanes* are particularly important: they correspond to global accelerated reference frames generalizing the global Galilei ones of the non-relativistic Newton mechanics. When we add the gauge fixings, which restrict the hypersurfaces to hyperplanes, the embedding gauge variables  $z^\mu(\tau, \vec{\sigma})$  are

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standard manifestly Lorentz covariant approach. This is the piece of information lacking in the traditional definition of the asymptotic states of Fock space as tensor products of free particles: in such a state an asymptotic particle may live in the absolute future of another one and this leads to the spurious solutions of the Bethe-Salpeter equation in the theory of relativistic bound states. Let us also remark that for relativistic particles this description requires the choice of the sign of the energy of each particle. For all these topics see Ref. [1].

reduced to only ten:  $z^\mu(\tau, \vec{\sigma}) = x_s^\mu(\tau) + b_r^\mu(\tau) \sigma^r$ . While the centroid  $x_s^\mu(\tau)$  (with arbitrary velocity  $\dot{x}_s^\mu(\tau)$ ) describes the origin of the 3-coordinates  $\vec{\sigma}$  on each hyperplane, the spatial triad  $b_r^\mu(\tau)$  together with the unit normal  $l^\mu = b_r^\mu$  form an orthonormal tetrad. However the  $\tau$ -independency of  $l^\mu$  reduces to three the independent degrees of freedom in  $b_r^\mu(\tau)$  (three Euler angles descing a rotating spatial reference frame). Only ten first class constraints survive and  $l^\mu = \text{const}$  is the gauge fixing for three of them.

Moreover, for every timelike configuration of the isolated system there is a foliation whose hyperplanes are intrinsically defined by the configuration itself: the one with the hyperplanes orthogonal to its conserved total 4-momentum. By definition this foliation defines the *rest frame* of the configuration. To select this foliation, the tetrad  $b_A^\mu$  has to be gauge fixed to coincide with the polarization vectors  $\epsilon_A^\mu(u(p_s))$  [ $u^\mu(p_s) = p_s^\mu / \sqrt{p_s^2}$ ], columns of the standard Wigner boost for timelike Poincare' orbits (for this reason these hyperplanes are named *Wigner hyperplanes*). After this fixation the only surviving canonical variables are

- i) a canonical but non-covariant variable  $\tilde{x}_s^\mu$  replacing the centroid  $x_s^\mu$  and the conjugate momentum  $p_s^\mu$ , weakly equal to the total 4-momentum  $P^\mu$  of the system;
- ii) canonical variables for the system living inside the Wigner hyperplanes, which are either Lorentz scalars or Wigner-covariant tensors.

Four first class constraints remain:

- i) one,  $\sqrt{p_s^2} - \mathcal{M} \approx 0$ , identifies the invariant mass of the isolated system as the mass associated with  $\tilde{x}_s^\mu$ ;
- ii) the other three,  $\vec{P} \approx 0$ , say that the Wigner hyperplanes are the rest frame.

The variable  $\tilde{x}_s^\mu$  is playing the role of a *decoupled point particle clock*, since it describes the relativistic *external* canonical non-covariant 4-center of mass of the isolated system and it is the only variable which breaks Lorentz invariance whichever is the system (*universality of the breaking*). Associated with it there is an *external* canonical realization of the Poincare' group, whose Lorentz boost generators induce Wigner rotations on the Wigner tensors living inside Wigner hyperplanes.

This *external* viewpoint describes the relativistic version of the separation of the center

of mass and defines the *internal* canonical variables for the system inside the Wigner hyperplanes. Due to the rest-frame constraints  $\vec{P} \approx 0$  three of these degrees of freedom define a *internal gauge* 3-center of mass in each Wigner hyperplane. If we eliminate it with three gauge fixings<sup>13</sup>, we remain with only *internal relative Wigner-covariant* canonical variables for the system. Then, if we eliminate the last constraint by identifying the parameter  $\tau$ , labeling the hyperplanes, with the rest-frame scalar time  $T_s = p_s \cdot \tilde{x}_s / \sqrt{p_s^2} = p_s \cdot x_s / \sqrt{p_s^2}$ , we obtain a new instant form of dynamics, the *rest-frame instant form*, with a decoupled external 4-center of mass and internal relative canonical variables. There is also a *unfaithful internal* canonical realization of the Poincare' group. This kinematical framework has allowed the definition of a new kinematics for the relativistic N-body problem [25], a study of relativistic rotational kinematics<sup>14</sup> and of multipole expansions [27]. The extension of this kinematics to relativistic perfect fluids and extended bodies is under investigation [28].

Then we looked for a formulation of general relativity with matter such that the switching off of the Newton constant  $G$  would produce the description of the same matter in the rest-frame instant form of parametrized Minkowski theory with the general relativistic general covariance deparametrizing to the special relativistic one.

We started with the following family of non-compact spacetimes [1,24,29,30]:

i) globally hyperbolic, so that the ADM Hamiltonian formulation is well defined if we start from the ADM action instead that from the Hilbert one;

ii) topologically trivial, so that they can be foliated with spacelike hypersurfaces diffeomorphic to  $R^3$  (3+1 splitting of spacetime with  $\tau$ , the scalar parameter labeling the leaves,

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<sup>13</sup>It is convenient to force it to coincide with the centroid  $x_s^\mu$ , origin of the 3-coordinates.

<sup>14</sup>Since non-relativistic concepts like Jacobi coordinates, relative masses and tensors of inertia are not extensible to the relativistic level, a new treatment of rotational kinematics for deformable bodies was needed. This has been found in Ref. [26], where *dynamical body frames* and *canonical spin bases* were defined and shown to be extensible to special relativity.

as a mathematical time);

iii) asymptotically flat at spatial infinity and with boundary conditions at spatial infinity independent from the direction, so that the *spi group* of asymptotic symmetries is reduced to the Poincare' group with the ADM Poincare' charges as generators <sup>15</sup>. In this way we can eliminate the *supertranslations*, which are the obstruction to define angular momentum in general relativity, and we have the type of boundary conditions which are needed to get well defined non-Abelian charges in Yang-Mills theory, opening the possibility of a unified description of the four interactions with all the fields belonging to same type of function space. All these requirements imply that the *allowed foliations* of spacetime must have the spacelike hyperplanes tending in a direction-independent way to Minkowski spacelike hyperplanes at spatial infinity, which moreover must be orthogonal there to the ADM 4-momentum. But these are the conditions satisfied by the singularity-free Christodoulou-Klainermann spacetimes [31], in which the allowed hypersurfaces define the *rest frame of the universe* and naturally become the Wigner hyperplanes when  $G \mapsto 0$ . Therefore there are *preferred asymptotic inertial observers*, which can be identified with the *fixed stars*. These allowed hypersurfaces have been called *Wigner-Sen-Witten hypersurfaces*, because it can be shown that the Frauendiener reformulation [32] of Sen-Witten equations for triads allows (after the restriction to the solutions of Einstein's equations) to transport the asymptotic tetrads of the inertial observers in each point of the hypersurface, generating a *local compass of inertia* to be used to define *rotations with respect to the fixed stars* <sup>16</sup>.

iv) All the fields have to belong to suitable weighted Sobolev spaces so that the allowed spacelike hypersurfaces are Riemannian 3-manifolds without Killing vectors: in this way we

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<sup>15</sup>When we switch off the Newton constant  $G$ , the ADM Poincare' charges in adapted coordinates become the generators of the internal Poincare' group of parametrized Minkowski theories.

<sup>16</sup>Instead the standard Fermi-Walker transport of the tetrads of a timelike observer is a standard of non-rotation with respect to a local observer in free fall.

avoid the analogue of the Gribov ambiguity in general relativity and we can get a unification of the function spaces of gravity and particle physics.

After all these preliminaries it is possible to study the Hamiltonian formulation of both ADM metric [24] and tetrad [29,30] gravity <sup>17</sup> with their (8 and 14 respectively) first class constraints as generators of the Hamiltonian gauge transformations. Then it is possible to look, at least at a heuristic level, for Shanmugadhasan canonical transformations performing the division between the gauge variables and the Dirac observables for the gravitational field. A complete exposition of these topics is in Refs. [24,29,30], where it is shown that it is possible to define a *rest frame instant form of gravity* in which the effective Hamiltonian for the evolution is the *ADM energy*  $E_{ADM}$  <sup>18</sup>.

Let us consider ADM canonical metric gravity. After an allowed 3+1 splitting of space-time with spacelike hypersurfaces  $\Sigma_\tau$ , the 4-metric  ${}^4g_{AB}(\tau, \vec{\sigma})$  in adapted coordinates is replaced with the lapse  $N(\tau, \vec{\sigma})$  and shift  $N_r(\tau, \vec{\sigma})$  functions and with the 3-metric  ${}^3g_{rs}(\tau, \vec{\sigma})$  on  $\Sigma_\tau$ . The conjugate momenta are  $\pi_N(\tau, \vec{\sigma})$ ,  $\pi_N^r(\tau, \vec{\sigma})$ ,  ${}^3\Pi^{rs}(\tau, \vec{\sigma})$ , respectively. There are four primary constraints  $\pi_N(\tau, \vec{\sigma}) \approx 0$ ,  $\pi_N^r(\tau, \vec{\sigma}) \approx 0$  and four secondary ones: the superhamiltonian constraint  $\mathcal{H}(\tau, \vec{\sigma}) \approx 0$  and the supermomentum constraints  $\mathcal{H}^r(\tau, \vec{\sigma}) \approx 0$ . Therefore there are *eight arbitrary gauge variables*, four of which are the lapse and shift functions. All

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<sup>17</sup>More natural for the coupling to the fermions of the standard model of particles. Moreover tetrad gravity is naturally a theory of timelike accelerated observers, generalizing the ones of parametrized Minkowski theories. In Refs. [29,30] there is a new parametrization of tetrad gravity, still utilizing the ADM action, which emphasizes this aspect and allows to solve 13 of the 14 constraints.

<sup>18</sup>The superhamiltonian constraint generates *normal* deformations of the spacelike hypersurfaces, which are *not* interpreted as a time evolution (like in the Wheeler-DeWitt approach) but as the Hamiltonian gauge transformations ensuring that the description of gravity is independent from the 3+1 splitting of spacetime like in parametrized Minkowski theories.

the constraints are first class and the Dirac Hamiltonian is <sup>19</sup>  $H_D = \int d^3\sigma \left( N(\tau, \vec{\sigma}) \mathcal{H}(\tau, \vec{\sigma}) + N_r(\tau, \vec{\sigma}) \mathcal{H}^r(\tau, \vec{\sigma}) + \lambda_N(\tau, \vec{\sigma}) \pi_N(\tau, \vec{\sigma}) + \lambda_{N_r}(\tau, \vec{\sigma}) \pi_{N_r}^r(\tau, \vec{\sigma}) \right) + (\text{surface terms})$ . The arbitrary functions  $\lambda_N(\tau, \vec{\sigma})$ ,  $\lambda_{N_r}(\tau, \vec{\sigma})$  are the Dirac multipliers, which are the source of the indeterminism in the Hamilton equations. The Hamiltonian version [33] of the *hole argument* amounts to fix completely the gauge freedom of the Dirac multipliers and of the lapse and shift functions outside the hole  $\mathcal{A}$ , but leaving them arbitrary inside the hole.

As shown in Ref. [24] the correct procedure to add the gauge fixings is the following:

i) add three gauge fixings  $\chi_r(\tau, \vec{\sigma}) \approx 0$  to the secondary supermomentum constraints: this amounts to a choice of 3-coordinates on  $\Sigma_\tau$ . The requirement of time constancy of the constraints  $\chi_r(\tau, \vec{\sigma}) \approx 0$  will generate three gauge fixings  $\varphi_r(\tau, \vec{\sigma}) \approx 0$  for the primary constraints  $\pi_{N_r}^r(\tau, \vec{\sigma}) \approx 0$ , which determine the shift functions  $N_r(\tau, \vec{\sigma})$  (and therefore the gravitomagnetic aspects and the eventual anisotropy of light propagation). The time constancy of the  $\varphi_r$ 's will determine the Dirac multipliers  $\lambda_{N_r}$ 's.

ii) add a gauge fixing  $\chi(\tau, \vec{\sigma}) \approx 0$  to the secondary superhamiltonian constraint, which determines the form of the spacelike hypersurface  $\Sigma_\tau$  (it is a statement about its extrinsic curvature). Its time constancy produces the gauge fixing  $\varphi(\tau, \vec{\sigma}) \approx 0$  for the primary constraint  $\pi_N(\tau, \vec{\sigma}) \approx 0$ , which determines the lapse function  $N(\tau, \vec{\sigma})$ , i.e. how the surfaces  $\Sigma_\tau$  are packed in the foliation. Now the 3+1 splitting of spacetime is completely determined and the time constancy of  $\varphi(\tau, \vec{\sigma}) \approx 0$  determines the Dirac multiplier  $\lambda_N(\tau, \vec{\sigma})$ . A posteriori after having solved the Hamilton equations one could find the embedding  $z^\mu(\tau, \vec{\sigma})$  of these Wigner-Sen-Witten hypersurfaces into the spacetime.

At this stage the canonical reduction is completed by going to Dirac brackets, the Dirac Hamiltonian reduces to the surface term, which can be shown [24] to be equivalent to the ADM energy. Therefore it becomes the effective Hamiltonian for the gauge invariant observables parametrizing the reduced phase space.

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<sup>19</sup>As shown in Ref. [24] the surface terms involve the ADM Poincare' charges.

To find a canonical basis of Dirac observables for the gravitational field in absence of known solutions of the superhamiltonian constraint, we can perform a quasi-Shanmugadhasan canonical transformation adapted to only seven of the constraints and utilize the information (see Ref. [24] for its justification) that this constraint has to be interpreted as the *Lichnerowicz equation* for the conformal factor  $\phi(\tau, \vec{\sigma}) = (\det {}^3g(\tau, \vec{\sigma}))^{1/12} = e^{q(\tau, \vec{\sigma})/2}$  of the 3-metric. The result is ( $a = 1, 2$ )

$$\begin{array}{|c|c|c|} \hline N & N_r & {}^3g_{rs} \\ \hline \pi_N \approx 0 & \pi_N^r \approx 0 & {}^3\tilde{\Pi}^{rs} \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|c|c|c|} \hline N & N_r & \xi^r & \phi & r_{\bar{a}} \\ \hline \approx 0 \approx 0 & \approx 0 & \pi_\phi & \pi_{\bar{a}} & \\ \hline \end{array} . \quad (0.1)$$

where  $\xi^r(\tau, \vec{\sigma})$  are a parametrization of the group manifold of the passive 3-diffeomorphisms of  $\Sigma_\tau$ , describing its changes of 3-coordinates. The Hamiltonian *gauge variables* are the seven configuration variables  $N(\tau, \vec{\sigma})$ ,  $N_r(\tau, \vec{\sigma})$ ,  $\xi^r(\tau, \vec{\sigma})$  (they depend on the 4-metric and its space gradients) and the momentum  $\pi_\phi(\tau, \vec{\sigma})$  conjugate to the conformal factor (it depends also on the time derivative of the 4-metric). The variables  $\xi^r(\tau, \vec{\sigma})$  and  $\pi_\phi(\tau, \vec{\sigma})$  can be thought as a possible 4-coordinate system with the Lorentz signature given by the pattern "3 configuration + 1 momentum" variables.

The physical deterministic degrees of freedom of the gravitational field are the non-local <sup>20</sup> Dirac observables (their expression in terms of the original variables is not known)  $r_a(\tau, \vec{\sigma})$ ,  $\pi_a(\tau, \vec{\sigma})$ ,  $a = 1, 2$ , which in general are not Bergmann observables being non-tensorial and coordinate-dependent. Even if we do not know the solution  $\phi = \phi[\xi^r, \pi_\phi, r_a, \pi_a]$  of the Lichnerowicz equation, the class of Hamiltonian gauges defined by the gauge fixing  $\chi(\tau, \vec{\sigma}) = \pi_\phi(\tau, \vec{\sigma}) \approx 0$  has the special property that the Dirac observables  $r_a(\tau, \vec{\sigma})$ ,  $\pi_a(\tau, \vec{\sigma})$  remain canonical also at the level of Dirac brackets. It is possible to study how to solve all the other constraints (also in tetrad gravity [30]) and how to express all the original

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<sup>20</sup>This non-locality can be considered a manifestation of *Mach's principle*: the knowledge of the full 3-space at each time is needed to determine the physics in the local neighborhood of each point of spacetime.



variables in terms of the Dirac observables associated to a chosen gauge. This allows for the first time to arrive at a completely fixed Hamiltonian gauge of the 3-orthogonal type <sup>21</sup>, which when restricted to the solutions of Einstein's equations (i.e. *on-shell*) is equivalent to a well defined choice of 4-coordinates for spacetime. It is now under investigation how to find a post-Minkowskian approximation to Einstein spacetimes based on the linearization of the theory in this completely fixed Hamiltonian gauge so to make contact with the theory of gravitational waves.

It is evident that the Hamiltonian gauge variables of canonical gravity carry an information about observers in spacetime, so that they are not inessential variables like in electromagnetism and Yang-Mills theory but take into account the fact that in general relativity global inertial reference frames do not exist <sup>22</sup>.

The separation between gauge variables and Dirac observables is an extra piece of (non-local) information [10,33], which has to be added to the equivalence principle, asserting the local impossibility to distinguish gravitational from inertial effects, to visualize which of the local forces acting on test matter are *generalized inertial (or fictitious) forces depending on the Hamiltonian gauge variables* and which are *genuine gravitational forces depending on the Dirac observables, which are absent in Newtonian gravity* <sup>23</sup>. Both types of forces

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<sup>21</sup>Namely with  ${}^3g_{rs}(\tau, \vec{\sigma})$  diagonal and with  ${}^3g_{rr}(\tau, \vec{\sigma}) = f_r(r_a(\tau, \vec{\sigma}))$ . The 3-orthogonal class of gauges seems to be the nearest one to the physical laboratories on the Earth. Let us remember that the standards of length and time are coordinate units and not Bergmann observables [34].

<sup>22</sup>The equivalence principle only allows the existence of local inertial frames along timelike geodesics describing the worldline of a scalar test particle in free fall.

<sup>23</sup>When we will introduce dynamical matter, this Hamiltonian procedure will lead to distinguish among action-at-a-distance, gravitational and apparent effects. It will be important to see the implications on concepts like gravitational passive and active masses and more in general on the problem of the origin of inertia and its connection with the various formulations of the Mach

have a *different appearance in different 4-coordinate systems*. In every 4-coordinate system (on-shell completely fixed Hamiltonian gauge)

i) the *genuine tidal gravitational forces* in the geodesic deviation equation will be well defined gauge-dependent functionals only of the Dirac observables associated to that gauge, so that Dirac's observables can be considered *generalized non-local tidal degrees of freedom* ;

ii) the geodesics will have a different geometrical form which again is functionally dependent on the Dirac observables in that gauge;

iii) the description of the relative 3-acceleration of a free particle in free fall given in the local rest frame of an observer will generated various terms identifiable with the general relativistic extension of the non-relativistic inertial accelerations and again these terms will depend on both the Dirac observables and the Hamiltonian gauge variables <sup>24</sup>.

Therefore the Hamiltonian gauge variables, which change value from a gauge to another one, *describe the change in the appearance* of both the physical and apparent gravitational forces going (on-shell) from a coordinate system to another one. This is similar to what happens with non-relativistic inertial forces, which however describe only apparent effects due to the absence of genuine dynamical degrees of freedom (Dirac observables) in Newtonian gravity. At the non-relativistic level, Newtonian gravity is described only by action-at-a-distance forces and, in absence of matter, there are *no tidal* forces among test particles, since they are determined by the variation of the action-at-a-distance force on the test particle

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principle.

<sup>24</sup>See the local interpretation [35] of *inertial* forces as effects depending on the *choice of a congruence of time-like observers with their associated tetrad fields* as a reference standard for their description. In metric gravity these tetrad fields are used only to rebuild the 4-metric. The real theory taking into account all the properties of the tetrad fields is *tetrad gravity*. Note that the definition of gravito-magnetism as the effects induced by  ${}^4g_{\tau r}$  is a pure inertial effect, because it is determined by the shift gauge variables.

created by the Newton potential of a massive body. Instead in vacuum general relativity the geodesic deviation equation shows that tidal forces, locally described by the Riemann tensor, are acting on test particles also in absence of every kind of matter: on-shell, in any chosen 4-coordinate system, these tidal forces are functionals of the non-local Dirac observables of the gravitational field in the completely fixed Hamiltonian gauge which corresponds on-shell to the chosen 4-coordinates. Independently of gravity, in Newtonian physics we speak of *global inertial (or fictitious) forces proportional to the mass* when we look at matter not from an inertial reference frame <sup>25</sup> but from an accelerated Galilean reference frame <sup>26</sup>. If the non-inertial reference frame has translational acceleration  $\vec{w}(t)$  and angular velocity  $\vec{\omega}(t)$  with respect to a given inertial frame, a particle with free motion ( $\vec{a} = \ddot{\vec{x}} = 0$ ) in the inertial frame has the following acceleration in the non-inertial frame

$$\vec{a}_{NI} = -\vec{w}(t) + \vec{x} \times \dot{\vec{\omega}}(t) + 2\dot{\vec{x}} \times \vec{\omega}(t) + \vec{\omega}(t) \times [\vec{x} \times \vec{\omega}(t)]. \quad (0.2)$$

After multiplication of this equation by the particle mass, the second term on the right hand side is the *Jacobi force*, the third term the *Coriolis force* and the fourth one the *centrifugal force*.

In Ref.37 a description, generally covariant under arbitrary passive Galilean coordinate transformations [ $t' = T(t)$ ,  $\vec{x}' = \vec{f}(t, \vec{x})$ ], of a free particle was given. The analogue of Eq.(0.2) contains more general *apparent* forces, which reduce to those in Eq.(0.2) in particular rigid coordinate systems.

While in Newtonian physics an absolute reference frame is an imagined extension of a rigid body and a clock (with any coordinate systems attached), in general relativity we must replace the rigid body either by a cloud of test particles in free fall (geodesic congruence) or

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<sup>25</sup>See Ref. [36] for the determination of *quasi-inertial reference frames* in astronomy as those frames in which rotational and linear acceleration effects are under the sensibility threshold of the measuring instruments.

<sup>26</sup>With arbitrary global translational and rotational 3-accelerations

by a test fluid (non-geodesic congruence for non-vanishing pressure).

Therefore in general relativity, where there are no global inertial reference frames, we have to use either a single accelerated time-like observer or a congruence of accelerated time-like observers with an associated conventionally chosen either tetrad or field of tetrads. Usually this is done by introducing *test observers* which describe the phenomena from their kinematical point of view without introducing any (either action or reaction) dynamical effect on the system (gravitational field plus dynamical matter).

In the case of a single test observer with his tetrad, see Ref. [33], after the choice of the local Minkowskian system of (Riemann-Gaussian) 4-coordinates where the line element becomes  $ds^2 = -\delta_{ij}dx^i dx^j + 2\epsilon_{ijk}x^j \frac{\omega^k}{c} dx^o dx^i + (1 + \frac{2\vec{a}\cdot\vec{x}}{c^2}(dx^o)^2)$  (the constants  $\vec{a}$  and  $\vec{\omega}$  are constant functionals of the Dirac observables of the gravitational field in this particular gauge), the test observer describes a nearby time-like geodesics  $y^\mu(\lambda)$  ( $\lambda$  is the affine parameter or proper time) followed by a test particle in free fall with the following spatial equation:  $\frac{d^2\vec{y}}{(dy^o)^2} = -\vec{a} - 2\vec{\omega} \times \frac{d\vec{y}}{dy^o} + \frac{2}{c^2}(\vec{a} \cdot \frac{d\vec{y}}{dy^o}) \frac{d\vec{y}}{dy^o}$ . If the test observer is in free fall (geodesic observer) we have  $\vec{a} = 0$ . If the triad of the test observer is Fermi-Walker transported (standard of non-rotation of the gyroscope) we have  $\vec{\omega} = 0$ .

Therefore the relative acceleration of the particle with respect to the observer with this special system of coordinates (replacing the global non-inertial non-relativistic reference frame) is composed by the observer 3-acceleration plus a relativistic correction and by a Coriolis acceleration. With other coordinate systems, other terms would appear. These are the inertial effects due to the Hamiltonian gauge variables.

In conclusion different on-shell Hamiltonian gauge fixings, corresponding to on-shell variations of the Hamiltonian gauge variables, give rise to *different appearances* of the physical forces as gauge-dependent functionals of the Dirac observables in that gauge of the type  $F(r_{\vec{a}}, \pi_{\vec{a}})$  (like  $\vec{a}$  and  $\vec{\omega}$  in the previous example). Newtonian gravity is recovered with a double limit:

i) Zero curvature limit, which is obtained by sending to zero the Dirac observables. In this way we get Minkowski space-time (a solution of Einstein's equations) with those systems

of coordinates which are compatible with Einstein's theory. As shown in Refs. [24,30] this implies the vanishing of the Cotton-York 3-conformal tensor, namely that the allowed 3+1 splittings of Minkowski space-time compatible with Einstein's equations have the *leaves 3-conformally flat* in absence of matter.

ii) The  $(c \rightarrow \infty)$  limit.

This implies that these functionals must be rewritten as the limit for vanishing Dirac observables of series in  $1/c$  :  $F_{newton} = \lim_{c \rightarrow \infty} \lim_{r_{\bar{a}}, \pi_{\bar{a}} \rightarrow 0} \left( F_o + \frac{1}{c} F_1 + \dots \right) = F_o|_{r_{\bar{a}}=\pi_{\bar{a}}=0}$ .  $F_{newton}$ , which may be coordinate dependent, becomes the *Newtonian inertial force* in the corresponding general Galilean coordinate system.

Let us come back to the problem of the physical identification of the mathematical points as point-events [10,33], namely as physical events. As shown in Ref. [39] there are 14 algebraically independent curvature scalars for  $M^4$ , which are reduced to four when Einstein equations without matter are used. Bergmann and Komar [40] discovered that the four (curvature scalars, i.e. Bergmann observables) eigenvalues  $\lambda_i(\tau, \vec{\sigma})$ ,  $i = 1, \dots, 4$ , of the Weyl tensor do *not depend* on the lapse and shift functions and can be used to define systems of *intrinsic pseudo-4-coordinates*  $F^{[A]}(\lambda_i(\tau, \vec{\sigma}))$  (we put the index  $A$  between square brackets to denote the scalar character of the functions  $F^{[A]}$  under spacetime diffeomorphisms), where the  $F^{[A]}$ 's are arbitrary functions restricted by the condition  $\det |\{F^{[A]}, \mathcal{H}^B\}| \neq 0$ , where  $\mathcal{H}^B = (\mathcal{H}; \mathcal{H}^r) \approx 0$  are the secondary constraints. This means that an admissible system of four gauge fixing constraints (determining the gauge variables  $\xi^r$  and  $\pi_\phi$ ) is

$$\chi^A(\tau, \vec{\sigma}) = \sigma^A - F^{[A]}(\lambda_i(\tau, \vec{\sigma})) \approx 0. \quad (0.3)$$

Clearly these conditions break completely general covariance by identifying coordinates with scalar fields.

As already said, the time constancy of these gauge fixings determines the lapse and shift functions and then the Dirac multipliers, so that at the end on the solutions of Einstein's equations we get a unique 4-coordinate system  $\sigma^A$  for the mathematical 4-manifold  $M$ . But in this completely fixed gauge the Weyl scalars become gauge-dependent functions of

the Dirac observables in this gauge,  $\lambda_i(\tau, \vec{\sigma})|_G = f_i^{(G)}(r_a(\tau, \vec{\sigma}), \pi_a(\tau, \vec{\sigma}))$ . Therefore, every 4-coordinate system of the mathematical spacetime may be identified by means of *four dynamical individuating fields* (in the terminology of Stachel [8]) which depend only on the Dirac observables in that gauge. Mathematical points of spacetime are transformed in physical point-events by means of the four independent degrees of freedom of the gravitational field.

In a sense *point-events of spacetime and the vacuum gravitational field are synonymous*.

Then *test matter* has to be used to measure the gravitational field, following Ref. [41] with a material reference fluid employing the intrinsic pseudo-coordinates. We have an axiomatic theory of measurement employing test matter [42], but no real theory based on *dynamical matter* due to the absence of solutions describing spacetimes without symmetries and due to the difficulties in using dynamical point particles in general relativity since the ultraviolet divergencies on the worldlines are much worse than in electrodynamics. The practice of the laboratories on and around the Earth is to use post-Newtonian corrections to Newtonian gravity and/or special relativistic theory of measurement for the electromagnetic phenomena. Astrometry [34] looks for the materialization of a global non-rotating, quasi-inertial reference frame in the form of a fundamental catalogue of stellar positions and proper motions, while physics in the solar system employs a barycentric post-Newtonian reference frame. Therefore, regarding matter, general relativity is essentially a dualistic theory. The situation becomes worse in the attempts of quantization: the need that the mass of test matter must be big to simulate a classical measuring apparatus contradicts its being test and not dynamical matter. There is a conflict between the role of mass as the charge of gravity with all the implications of the equivalence principle and quantum theory: it is a complete mystery which is the genesis of mass and why it seems not to be quantized.

Let us remark that at the level of Dirac brackets there is an induced non-commutative structure added to spacetime by the functions  $F^{[A]}$ .

This is the final theoretical answer to the problem raised by the *hole argument*, which

however employs a division between generalized inertial (the gauge variables) and tidal (the Dirac observables) effects in which both types of effects have a coordinate-dependent appearance, i.e. they are not Bergmann observables.

Bergmann and Komar [39,43] also defined an *intrinsic (tetradic-like) 4-metric*

$${}^4g_{[A][B]}(F^{[E]}) = \frac{\partial \sigma^C}{\partial F^{[A]}} \frac{\partial \sigma^D}{\partial F^{[B]}} {}^4g_{CD}(\sigma^E), \quad (0.4)$$

whose ten components are Bergmann observables, but did not develop its implications.

The concept of an intrinsic 4-metric together with the (at least local) existence of the quasi-Shanmugadhasan canonical transformation leads to the following *conjecture*: there should exist a class of quasi-Shanmugadhasan canonical transformations such that both the resulting Hamiltonian gauge variables (the generalized inertial effects) and Dirac observables (the generalized tidal effects) are *also* Bergmann observables.

If the conjecture is true, there is a *preferred* family of canonical bases in the ADM phase space worthy of investigation both for trying to solve the superhamiltonian constraint and for a new attempt to the canonical quantization of gravity, in which only the physical (but Bergmann observable) degrees of freedom of the gravitational field, and not the inertial effects, are quantized, preserving in this way the causal structure of spacetime.

Let us remark that neither string theory nor loop quantum gravity have developed a strategy for finding the solution of the superhamiltonian constraint. As a consequence, we do not know the modifications of Newton law between two bodies at short distances (but which ones at the classical level? Planck length is a quantum effect depending on  $\hbar$ ) induced by Einstein's general relativity, since it depends on the conformal factor of the 3-metric which has to be found as the solution of Lichnerowicz equation.

Let us conclude with a comment on the possibility to arrive at an *operational definition of a region of spacetime* by means of the technological developments connected with the Global Positioning System (GPS) [44]. In Ref. [10] there is the description of GPS and of a possible modified (i.e. not using the gravitational field of the Earth as input) experimental setup and protocol for positioning and orientation, which should allow a physical individuation of

point-events in regions with non weak fields like near the Sun or Jupiter (see Refs. [45] for other proposals). One should employ a net of satellites to establish a *radar-gauge system of 4-coordinates*  $\sigma_{(R)}^A = (\tau_{(R)}; \sigma_{(R)}^r)$  in a finite region ( $\tau_{(R)} = \text{const}$  defines the radar simultaneity surfaces). Each satellite may be thought as a timelike observer (the satellite worldline) with a tetrad (the timelike vector is the satellite 4-velocity and the spatial triad is built with gyroscopes) and one of them is chosen as the origin of the radar-4-coordinates. Either by using test polarized light to measure the relative spatial rotation of triads or by measuring the motion of  $n \geq 4$  test particles [46] it should be possible to measure the components of the 4-metric in these radar-gauge coordinates. Then it is a matter of computation to check:

- i) whether Einstein's equation in radar-gauge coordinates are satisfied;
- ii) which are the functions  $F^{[A]}$  to be used in Eqs.(0.3) to identify the radar-gauge by means of the intrinsic coordinate method.

This procedure would close the *coordinate circuit* of general relativity, linking individuation to experimentation [10].



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